Universidad de Navarra

Calculus I

Teaching guide 2025-26

INTRODUCTION

Brief description

The course Calculus focuses on the study and analysis of real-valued functions of a real variable and their properties. It is a fundamental basis for the development of advanced mathematical knowledge and its application in various scientific and technical disciplines.

During the course, different types of functions (rational, trigonometric, exponential, logarithmic...) and their properties are studied, and their behaviour is analysed in terms of limits, continuity and differentiability.

Additionally, the local analysis of functions is explored in greater depth, studying their maxima and minima, and concavity and convexity. Taylor's formula is introduced, which allows for the approximation of functions by a polynomial.

The course also includes the study of integrals and their properties, as well as numerical sequences and numerical and functional series, which are fundamental in mathematical analysis.

Qualification (Module / Subject)

- Industrial Technologies Engineering (Basic training / Basic mathematics)
- Mechanical Engineering (Basic training / Basic mathematics)
- Electrical Engineering (Basic training / Basic mathematics)
- Industrial Electronics Engineering (Basic training / Basic mathematics)
- Telecommunication Systems Engineering (Basic training / Basic mathematics)
- Industrial Organisation Engineering (Basic training / Basic mathematics)
- Industrial Design and Product Development Engineering (Basic training / Basic mathematics)
- Biomedical Engineering (Basic training / Basic mathematics)
- Artificial Intelligence Engineering (Basic training / Basic mathematics)

Details

- **ECTS:** 6 ECTS
- **Year, semester:** 1st year, 1st semester
- **Character:** Basic
- **Language:** English

Lecturers of the course

- Csiszár, Kara Orsolya / Profesora visitante
- de Zárraga Rodríguez, Marta / Profesora titular
- Sunsundegi Oiarzabal, Bruno / Personal investigador en formación

LEARNING OUTCOMES

Universidad de Navarra

INDUSTRIAL TECHNOLOGIES ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

R55 - Ability to solve problems involving numerical methods for differential equations and ability to apply mathematical optimisation techniques.

MECHANICAL ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

R24 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

ELECTRICAL ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

R3 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

INDUSTRIAL ELECTRONICS ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral

calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

R22 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

TELECOMMUNICATION SYSTEMS ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

INDUSTRIAL ORGANISATION ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial differential equations; numerical methods; numerical algorithms; statistics and optimisation.

R40 - Provide a solid foundation in science, technology, operations management, production and business management.

INDUSTRIAL DESIGN AND PRODUCT DEVELOPMENT ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R3 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; numerical methods; numerical algorithms; statistics and optimisation.

BIOMEDICAL ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R23 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; numerical methods; numerical algorithms; statistics and optimisation.

R31 - Training must provide the graduate with a solid scientific basis to enable them to rigorously address any professional challenges they may face in the biomedical sector.

ARTIFICIAL INTELLIGENCE ENGINEERING

R1 - Describe the fundamentals of linear algebra; geometry, differential geometry, differential and integral calculus, differential and partial differential equations, numerical methods, numerical algorithms, statistics and optimisation to solve mathematical problems applied to the field of engineering.

PROGRAM

1. The real line: Real numbers. Subsets of R. Bounded sets. Intervals and neighbourhoods. Distance and absolute value.

2. Limits and continuity: Real-valued functions of a real variable. Operations with functions. Transformation of function graphs. Limits of functions. Properties of limits. Operations with limits. Continuity of functions. Operations with continuous functions. Properties of continuous functions. Monotonic functions.

3. Elementary functions: Polynomial function. Rational function. Absolute value function. Trigonometric functions. Inverse functions of the trigonometric functions. Exponential function. Logarithmic function. Hyperbolic functions. Inverse functions of the hyperbolic functions. Power function. Infinitesimals and infinites.

4. Differentiability: Differentiability of real-valued functions of a real variable. Relationship between differentiability and continuity. Operations with differentiable functions. Differentials.

5. Local study of functions: Increasing and decreasing intervals. Local and absolute extrema. Theorems for the local study of functions (Rolle's theorem and mean value theorem). Curvature. Inflection points.

6. Taylor's formula: Taylor polynomial. Taylor's theorem. Theorems for the local study of functions with successive vanishing derivatives.

7. Indefinite integrals: Indefinite integral. Integration by substitution method. Integration by parts method. Integration of rational functions. Integration of functions reducible to rational functions.

8. Definite integrals: Riemann integral. Properties of the Riemann integral. Relationship between the Riemann integral and the primitives (first and second fundamental theorem of integral calculus). Integration by substitution method for definite integrals. Integration by parts method for definite integrals. Calculation of the area of plane regions.

9. Improper integrals: Integration of bounded functions on unbounded intervals. Integration of unbounded functions on bounded intervals. Integration of unbounded functions on unbounded intervals. Convergence tests for improper integrals of non-negative functions.

10. Numerical sequences: Numerical sequences. Classification of numerical sequences. Properties of convergent sequences. Properties of divergent sequences. Monotonic

sequences. Asymptotically equivalent sequences. Tests for the calculation of the limit of sequences.

11. Numerical and functional series: Numerical series. Geometric series. General harmonic series. Properties of numerical series. Series of positive terms. Absolutely convergent series. Alternating series. Functional series. Power series.

EDUCATIONAL ACTIVITIES

The 150-180 hours (6 ECTS) dedication to the course Calculus is divided into the following educational activities.

Theoretical lectures (20 hours) and practical lectures (40 hours)

The lectures (in the classroom) will encompass both the theoretical development of the course and the resolution of exercises.

Tutorials (1 hour)

The lecturers will be available to address any questions and provide students with more personalised support in acquiring knowledge and skills.

Independent study (111 hours)

Students will have to study the theory presented in each lecture independently and work on the exercises proposed by the lecturers. In order to facilitate the monitoring of the course and the independent study, students will have access to study material including proposed exercises, examples of previous exams, videos and tutorials.

Assessment (8 hours)

Throughout the semester, midterm exams will be conducted to evaluate students' progress, as well as a final exam.

ASSESSMENT

Notice: Please be reminded that any attempt at fraud, copying, plagiarism or other irregular behaviour constitutes a serious infraction and, as such, is subject to sanctions in accordance with the "System of rules on coexistence at the University of Navarra" (Title IV "Academic discipline regulations of students").

Ordinary call

The assessment will be based on the following activities.

- Midterm exam 1 (ME1):
  - Score: 2.5 points.
  - Date: 27th of September from 11:30 to 13:00.

- Midterm exam 2 (ME2):
  - Score: 3.5 points.
  - Date: 27th of October from 8:15 to 9:45.
- Final exam (FE):
  - Content: The final exam will have three parts (FE1, FE2 and FE3). In parts FE1 and FE2, the same contents as in the midterm exams ME1 and ME2 will be evaluated, respectively. In part FE3, the rest of the contents of the course will be evaluated.
  - Score: FE1 (2.5 points) + FE2 (3.5 points) + FE3 (3.5 points).
  - Date: 4th of December at 9:00.
- Continuous assessment (CA):
  - Content: The rest of evaluable activities carried out during the semester (weekly tests, Codex notebooks...).
  - Score: 0.5 points.

The grade of the ordinary call is obtained with the following formula:

max (ME1, FE1) + max (ME2, FE2) + FE3 + CA

In order to pass the course, the grade of the ordinary call must be greater than or equal to 5.

Extraordinary call

The assessment will be based on the extraordinary final exam, in which all the contents of the course will be evaluated. The grade of the extraordinary call is the mark obtained in the extraordinary final exam. In order to pass the course, the grade of the extraordinary call must be greater than or equal to 5. The marks obtained in the midterm exams, the final exam and the continuous assessment of the ordinary call will not be considered in the extraordinary call.

Date: 9th of January at 9:00.

BIBLIOGRAPHY AND RESOURCES

Basic bibliography

- Lecturers' notes.
- G. B. Thomas and R. L. Finney, *Calculus and analytic geometry*, 9th edition, Pearson India, 1996. Find it in the library
- A. García, F. García, A. Gutiérrez, A. López, G. Rodríguez and A. de la Villa, *Cálculo I. Teoría y problemas de análisis matemático en una variable*, 2nd edition, Clagsa, 1994. Find it in the library

Supplementary bibliography

- T. M. Apostol, *Calculus*, Vol. I, 2nd edition, John Wiley & Sons, 1967. Find it in the library
- T. M. Apostol, *Mathematical analysis*, 2nd edition, Addison-Wesley, 1974. Find it in the library
- W. Rudin, *Principles of mathematical analysis*, 3rd edition, McGraw-Hill, 1976. Find it in the library

Universidad de Navarra

OFFICE HOURS

The lecturers will organise a weekly question session in which they will be available to address any questions regarding the theory and the proposed exercises. The schedule and the location of the question session will be determined at the beginning of the course.

Additionally, the lecturers will be available for individual consultations. It is necessary to request an appointment via email in advance.