Universidad de Navarra

Calculus I

Teaching guide 2025-26

INTRODUCTION

Brief description

The course Calculus focuses on the study and analysis of real-valued functions of a real variable and their properties. It is a fundamental basis for the development of advanced mathematical knowledge and its application in scientific and technical disciplines.

During the course, different types of functions (rational, trigonometric, exponential and logarithmic) and their properties are studied, and their behaviour is analysed in terms of limits, continuity and differentiability.

Additionally, the local analysis of functions is explored in greater depth, studying their maxima and minima, and concavity and convexity. Taylor's formula is introduced, which allows for the approximation of functions by a polynomial.

The course also includes the study of definite integrals and their properties, as well as numerical sequences and numerical and functional series, which are fundamental in mathematical analysis.

Qualification (Module/Subject)

- Industrial Technologies Engineering (Basic Training/Mathematics and Computer Science)
- Mechanical Engineering (Basic Training/Mathematics and Computer Science)
- Electrical Engineering (Basic Training/Mathematics and Computer Science)
- Industrial Electronics Engineering (Basic Training/Mathematics and Computer Science)
- Telecommunication Systems Engineering (Basic Training/Mathematics and Computer Science)
- Industrial Management Engineering (Basic Training/Mathematics and Computer Science)
- Industrial Design and Product Development Engineering (Basic Training /Mathematics and Computer Science)
- Biomedical Engineering (Basic Training/ Mathematics and Computer Science)
- Artificial Intelligence Engineering (Basic Training/Basic Mathematics)

Details

- **ECTS:** 6 ECTS
- **Year, semester:** 1st year, 1st semester
- **Character:** Compulsory
- **Language:** English

Lecturers of the course

- Csiszár, Orsolya / Profesora visitante
- de Zárraga Rodríguez, Marta / Profesora titular
- Sunsundegi Oiarzabal, Bruno / Investigador en formación

Universidad de Navarra

LEARNING OUTCOMES (COMPETENCIES)

INDUSTRIAL TECHNOLOGIES ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

SC1 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial derivative equations; numerical methods; numerical algorithms; statistics and optimisation.

SC32 - Ability to solve problems involving numerical methods for differential equations and ability to apply mathematical optimisation techniques.

MECHANICAL ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

GC3 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

SC1 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial derivative equations; numerical methods; numerical algorithms; statistics and optimisation.

ELECTRICAL ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

GC3 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

SC1 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

INDUSTRIAL ELECTRONICS ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

GC3 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

SC1 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

TELECOMMUNICATION SYSTEMS ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study (Type: Knowledge or Content).

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation (Type: Knowledge or Contents).

INDUSTRIAL ORGANISATION ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study which builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

SC1 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

GC6 - Provide a solid foundation in science, technology, operations management, production and business management.

INDUSTRIAL DESIGN AND PRODUCT DEVELOPMENT ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

SC1 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; numerical methods; numerical algorithms; statistics and optimisation.

BIOMEDICAL ENGINEERING

BC1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

GC1 - Training must provide the graduate with a solid scientific basis to enable them to rigorously address any professional challenges they may face in the biomedical sector.

SC29 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; numerical methods; numerical algorithms; statistics and optimisation.

ARTIFICIAL INTELLIGENCE ENGINEERING

R1 - Describe the fundamentals of linear algebra; geometry, differential geometry, differential and integral calculus, differential equations and in partial derivatives, numerical methods, numerical algorithms, statistics and optimisation to solve mathematical problems applied to the field of engineering (Type: Knowledge or contents).

PROGRAM

1. The real line: Real numbers. Subsets of $\mathbb{R}$. Bounded sets. Intervals and neighbourhoods. Distance and absolute value.

2. Limits and continuity: Real-valued functions of a real variable. Operations with functions. Transformation of function graphs. Limits of real-valued functions of a real variable. Properties of limits. Operations with limits. Continuity of real-valued functions of a real variable. Operations with continuous functions. Properties of continuous functions. Monotonic functions.

3. Elementary functions: Polynomial function. Rational function. Absolute value function. Trigonometric functions. Inverse trigonometric functions. Exponential function. Logarithmic function. Hyperbolic functions. Inverse hyperbolic functions. Power function. Infinitesimals and infinites.

4. Differentiability: Differentiability of real-valued functions of a real variable. Relationship between differentiability and continuity. Operations with differentiable functions. Differentials.

5. Local study of functions: Increasing and decreasing. Local and absolute extrema. Theorems for the local study of functions (Rolle's theorem and mean value theorem). Curvature. Inflection points.

6. Taylor's formula: Taylor polynomial. Taylor's theorem. Theorems for the local study of functions with successive vanishing derivatives.

7. Indefinite integrals: Indefinite integral. Integration by substitution method. Integration by parts method. Integration of rational functions. Integration of functions reducible to rational functions.

8. Definite integrals: Riemann integral. Properties of the Riemann integral. Relationship between the Riemann integral and the primitives (first and second fundamental theorems of integral calculus). Integration by substitution method for definite integrals. Integration by parts method for definite integrals. Calculation of the area of plane regions.

9. Improper integrals: Integration of bounded functions on unbounded intervals. Integration of unbounded functions on bounded intervals. Integration of unbounded functions on unbounded intervals. Convergence tests for improper integrals of non-negative functions.

10. Numerical sequences: Numerical sequences. Classification of numerical sequences. Properties of convergent sequences. Properties of divergent sequences. Monotonic sequences. Asymptotically equivalent sequences. Tests for the calculation of the limit of sequences.

11. Numerical and functional series: Numerical series. Geometric series. General harmonic series. Properties of numerical series. Series of positive terms. Absolutely convergent series. Alternating series. Functional series. Power series.

EDUCATIONAL ACTIVITIES

The 150-180 hours (6 ECTS) commitment to the course Calculus are divided into the following educational activities.

Theoretical lectures (20 hours) and practical lectures (40 hours)

The lectures (in the classroom) will encompass both the theoretical development of the course and the resolution of exercises.

Tutorials (1 hour)

The lecturers will be available to address any questions and provide students with more personalised support in acquiring knowledge and skills.

Independent study (111 hours)

Students will have to study the theory presented in each lecture independently and work on the exercises proposed by the lecturers. To facilitate course monitoring and independent study, students will have access to study materials, including proposed exercises, examples of previous exams, videos and tutorials.

Assessment (8 hours)

Throughout the semester, midterm exams will be conducted to evaluate students' progress, along with a final exam.

ASSESSMENT

Ordinary call

The mark of the ordinary call is calculated based on the ordinary final exam and the midterm exams. These exams will assess both theoretical and practical contents.

Extraordinary call

It will consist in an exam in which all the contents of the course will be evaluated. The mark of the extraordinary call is the mark obtained in the aforementioned exam. The marks obtained during the semester will not be considered.

BIBLIOGRAPHY AND RESOURCES

Basic bibliography

- Lecturers' notes.
- G. B. Thomas and R. L. Finney, *Calculus and analytic geometry*, 9th edition, Pearson India, 1996. Find it in the library
- A. García, F. García, A. Gutiérrez, A. López, G. Rodríguez and A. de la Villa, *Cálculo I. Teoría y problemas de análisis matemático en una variable*, 2nd edition, Clagsa, 1994. Find it in the library

Supplementary bibliography

- T. M. Apostol, *Calculus*, Vol. I, 2nd edition, John Wiley & Sons, 1967. Find it in the library
- T. M. Apostol, *Mathematical analysis*, 2nd edition, Addison-Wesley, 1974. Find it in the library
- W. Rudin, *Principles of mathematical analysis*, 3rd edition, McGraw-Hill, 1976. Find it in the library

OFFICE HOURS

The lecturers will organise a weekly question session in which they will be available to address any questions regarding the exercises proposed in each chapter (indicated in the "Planning" section). The schedule and location of the question session will be determined at the beginning of the course.

Additionally, the lecturers will be available for individual consultations. It is necessary to request an appointment via email in advance.