Universidad de Navarra

Calculus II

Teaching guide 2026-27

PRESENTATION

Brief description:

Calculus II is a course that focuses on the study and application of concepts and techniques involving calculus in multidimensional spaces. Through this course, students acquire a deeper understanding of functions and operations in multivariable contexts. This course is a generalisation of differential and integral calculus in one variable to the Euclidean space R^n .

Qualification (Module/Subject):

- Industrial Technologies Engineering (Basic Training/ Basic Mathematics)
- Mechanical Engineering (Basic Training/ Basic Mathematics)
- Electrical Engineering (Basic Training/ Basic Mathematics)
- Industrial Electronics Engineering (Basic Training/ Basic Mathematics)
- Telecommunication Systems Engineering (Basic Training/ Mathematics and Computer Science)
- Industrial Management Engineering (Basic Training/ Basic Mathematicse)
- Industrial Design and Product Development Engineering (Basic Mathematics)
- Biomedical Engineering (Basic Training/ Basic Mathematics)
- Artificial Intelligence Engineering (Basic Training/Basic Mathematics)

Details:

- **ECTS**: 6 ECTS
- **Year, semester**: 1st year, 2nd semester
- **Character**: Basic
- **Language**: English

Lecturers in the subject:

- Diego González Torres / Senior Researcher
- Csiszár, Kara Orsolya / Visiting Professor

LEARNING OUTCOMES

INDUSTRIAL TECHNOLOGIES ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

R2- Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential and partial derivative equations; numerical methods; numerical algorithms; statistics and optimisation.

R55 - Ability to solve problems involving numerical methods for differential equations and ability to apply mathematical optimisation techniques.

MECHANICAL ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

R24 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

ELECTRICAL ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

R2 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

R3 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

INDUSTRIAL ELECTRONICS ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

R22 - Knowledge in basic and technological subjects, which enables them to learn new methods and theories, and provides them with the versatility to adapt to new situations.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

TELECOMMUNICATION SYSTEMS ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study. (Type: Knowledge or content)

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation. (Type: Knowledge or contents)

INDUSTRIAL ORGANISATION ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects involving knowledge deriving from the cutting edge of their field of study.

R2 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; differential equations and in partial derivatives; numerical methods; numerical algorithms; statistics and optimisation.

R40 - Provide a solid foundation in science, technology, operations management, production and business management.

INDUSTRIAL DESIGN AND PRODUCT DEVELOPMENT ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

R3 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; numerical methods; numerical algorithms; statistics and optimisation.

BIOMEDICAL ENGINEERING

R1 - Students will have demonstrated knowledge and understanding in an area of study that builds on the foundation of general secondary education, and is usually at a level which, while relying on advanced textbooks, also includes some aspects that involve knowledge deriving from the cutting edge of their field of study.

R31 - Training must provide the graduate with a solid scientific basis to enable them to rigorously address any professional challenges they may face in the biomedical sector.

R23 - Ability to solve mathematical problems that may arise in engineering. Ability to apply knowledge of: linear algebra; geometry; differential geometry; differential and integral calculus; numerical methods; numerical algorithms; statistics and optimisation.

ARTIFICIAL INTELLIGENCE ENGINEERING

R1 - Describe the fundamentals of linear algebra; geometry, differential geometry, differential and integral calculus, differential equations and in partial derivatives, numerical methods, numerical algorithms, statistics and optimisation to solve mathematical problems applied to the field of engineering (Type: Knowledge or contents).

COURSE CONTENT

1. The space R^n: The Euclidean space R^n. Distance and balls in R^n. Open and closed sets.

2. Limits and continuity of functions of several variables: Scalar functions of n variables: domain, graph, level sets. Limits and continuity of scalar functions. Properties of continuous functions. Vector functions of dimension m of n variables. Limits and continuity of vector functions. Continuous function composition.

3. Differentiation between functions of several variables; Partial derivatives. Gradient vector. Higher order partial derivatives. Schwarz's theorem. Differentiable scalar functions. Properties of differentiable functions. Differential of a scalar function. Second differential of scalar functions. Hessian matrix. Differentiable vector functions. Jacobian matrix. Differential of a vector function. Second differential of vector functions. Composition of differentiable functions. Chain rule.

4. Differential operators: Gradient of a scalar field and its properties. Laplacian of a scalar field. Divergence and rotational of a vector field. Differential forms and potential functions.

5. Taylor's formula: Taylor's theorem and mean value theorem.

6. Implicit functions: Implicit scalar and vector functions.

7. Extremes of scalar functions: Absolute and relative extrema. Necessary conditions and sufficient conditions of relative extrema.

8. Conditional extrema: Lagrange multiplier method.

9. Parameter-dependent integrals: Definitions. Properties. Gamma and Beta functions.

10. Multiple integrals: Definitions. Calculation of double integrals. Change in variables. Triple integrals.

11. Curvilinear integrals: Plane curves and curves in space. Curvilinear integral of a scalar field. Curvilinear integral of a vector field. Conservative fields.

12. Surface integrals: Warped surfaces. Surface integral of a scalar field. Surface integral of a vector field.

13. Integral theorems: Green's theorem. Gauss's theorem. Divergence theorem. Stokes' theorem.

TRAINING ACTIVITIES

150-180h (6 ECTS) are set aside for Calculus II and are divided into the following training activities:

Theoretical face-to-face classes: 20 hours

Face-to-face practical classes, laboratories and workshops: 40 hours

Tutorials: 1 hour

Personal study: 112 hours

Evaluation: 7 hours

TEACHING METHODOLOGIES

- Expository classes
- Personal interview with the lecturer in a subject

- Student study based on different sources of information
- Evaluated tests

Face-to-face classes (in the classroom) will include both the **theoretical development of the** subject and the resolution of **practical exercises**. Study material (in the internal area) is made available to students for the purpose of monitoring the subject and personal study geared to the acquisition of knowledge, concepts and skills. The study material includes proposed exercises that need to be completed, solutions to the exercises, examples of exams from previous years, videos and tutorials, etc.

Lecturers will be available to **answer students' questions** and help them, in a more personalised way, to acquire knowledge, skills and abilities.

During the four-month period there will be different **tests to** evaluate the progress of the students, as well as a final exam on the date indicated in the timeline (internal area).

EVALUATION

Notice: Please be reminded that any attempt at fraud, copying, plagiarism or other irregular behaviour constitutes a serious infraction and, as such, is subject to sanctions in accordance with the "System of rules on coexistence at the University of Navarra" (Title IV "Academic discipline regulations of students").

ORDINARY CALL

Ordinary evaluation of the course will be carried out over three exams.

Midterm exam 1 (M1)

- Content: Approximately the material seen up to the day of the exam (to be specified well in advance).
- Score: 3 points.
- Date: Monday, 2nd of March at 8:30.

Midterm exam 2 (M2)

- Content: Approximately the material seen from the previous exam up to the day of the exam (to be specified well in advance).
- Score: 3 points.
- Date: Saturday, 11th of April.

Ordinary examination session

- Date: Monday, 25th of May at 9:00
- The exam will consist of three (F1, F2 and F3). The first two parts (F1 and F2) will cover the same contents as the midterm exams. The content of the third part (F3) will cover the rest of the contents.
- Score: 3 points (F1) +3 points (F2) + 4 points (F3).

Final mark will be: max(M1, F1) + max(M2, F2) + F3

Students must obtain at least 5 points in the final mark in order to pass the course.

EXTRAORDINARY CALL

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There will be a single exam covering all the contents of the course which will account for 100% of the mark. Marks obtained in the midterm exams of the ordinary call will not be considered in the extraordinary call. Students must obtain at least 5 points over 10 in order to pass the course.

Date: Tuesday, 23th of June at 9:00.

OFFICE HOURS

The question session will be on **Tuesdays from 16:30 to 18:00 in classroom 6**.

Students can also contact the teachers of the course by e-mail.

BIBLIOGRAPHY

Basic bibliography:

1. JM Martínez-Esnaola, C Blanco del Prado. Calculus II: Lecture Notes. School of Engineering, TECNUN, University of Navarra (2025).
2. A García, A López, F García, G Rodríguez, A Gutiérrez, A de la Villa. Calculo II: Teoría y problemas de funciones de varias variables. Glagsa, Madrid (2002). Find it in the library
3. J Stewart. Multivariable calculus. Thomson, Madrid (2002). Find it in the library

Supplementary bibliography:

1. J De Burgos Román. Infinitesimal Calculus: Theory and Problems. Alhambra (1992). Find it in the library
2. J De Burgos Román. Integral calculus (one and several variables): 70 useful problems. García-Maroto Editores (2007).
3. GB Thomas, RL Finney. Calculus with Analytic Geometry, Vols. I and II. Addison-Wesley Iberoamericana, Wilmington (1987). Find it in the library
4. W Rudin. Principles of mathematical analysis. McGraw-Hill, New York (1976). Find it in the library